Regge Poles

Gell-Mann Zachariasen & Frautschi

Regge - N.C. 1959
1960

Regge - Bottino, Longoni - Potential scat. for complex energy & angular
Pregni momenta

phys. idea ✓ Chew & Frautschi I P.R.L. Nov 15
✓ II Jan 1
pretentious

Math. techniques → Chew, Frautschi & Mandelstam (preprint)

Mandelstam, preprint

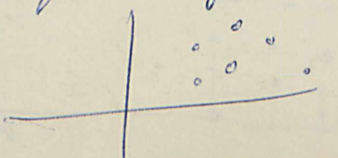
Lovelace I - Diffraction scat. & Mandelstam (preprint)
→ Local II. Harwell talk.
Wong, preprint
Udgaonkar Jan 15 P.R.L. (practical)
Gell-Man, Zachariasen & Frautschi.

$$z = \cos\theta$$

scat. amplitude

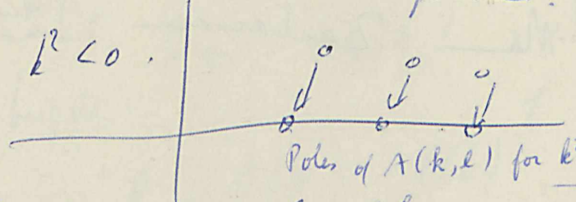
$$A(k, z) = \sum (2l+1)\, A(k, l)\, P_l(\cos\theta)$$

Regge → shows that in Pot'l scat. (Yukawa pots) that $A(k, l)$ has poles in complex l plane for $k^2 > 0$



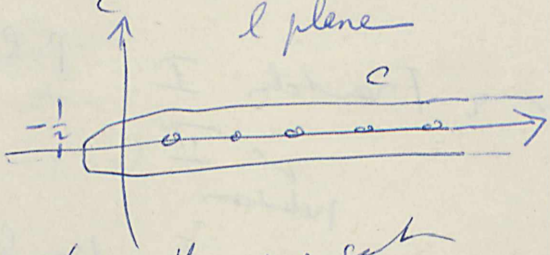
l plane

If $k^2 < 0$ (bound states) the poles are on real axis

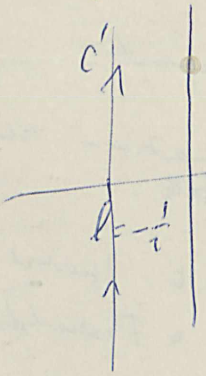


extend A to complex values of l.

$$A(k,z) = \frac{1}{i}\int_C dl\,(2l+1)\,A(k,l)\,\frac{P_l(-z)}{\sin\pi l}$$



If contour is deformed to C', then we get



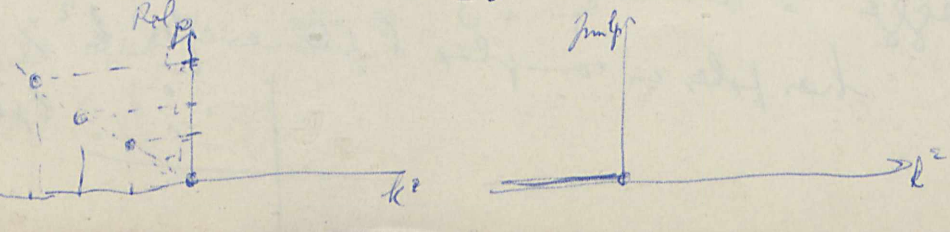
a additional term (contribution of pole terms)

$$A(k,z) = \frac{1}{i}\int_{C'} \quad + \sum_{\mathrm{Re}\, l_p > -\frac{1}{2}} \frac{(2l_p+1)\, a_{l_p}(k)\, P_{l_p}(-z)}{\sin\pi l_p(k)}$$

Meaning of this formula:

$l_p(k)$ $k^2 > 0$ l_p complex number

$k^2 < 0$ $\mathrm{Im}\, l_p = 0$ $\mathrm{Re}\, l_p$ = integer



Take case $k^2 < 0$.

In general $E_B = E_B(l)$ is a fn of angular momentum

energy of bound state l

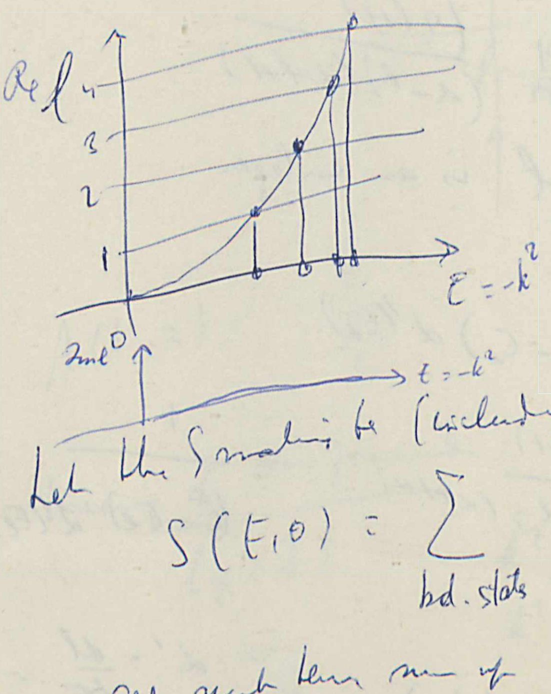
or $l = l(E) = l(k)$.

Hydrogen atom: let $l = j + \frac{1}{2}$

$$E = \frac{mc^2}{\sqrt{1 + \frac{\alpha^2}{(j+\frac{1}{2})^2}}}$$

$$l^2 = \frac{E^2}{m^2 - E^2}$$

integer values of l give the bound states



This is called trajectory of a pole in l–E plane.

Let the S matrix be (including contribution of bound states)

$$S(E,\theta) = \sum_{\text{bd. states}} \frac{P_l(\cos\theta)}{E - E_l}\,\alpha(E,l) \quad + \text{Rest}$$

All such terms sum up to a single term $\sum \dfrac{(2l_p+1)\,a_{l_p}(k)\,P_{l_p}(-z)}{\sin \pi l_p(k)}$

Chew Frautschi Mandelstam

$$l^2(E) = \frac{E^2}{m^2 - E^2}$$

$$A(k,z) = \text{Background} + \sum_{\mathrm{Re}\, l_p > -\frac{1}{2}} \frac{(2l_p+1)\, a_{l_p}(k)\, P_{l_p}(z)}{\sin \pi l_p(k)}$$

let $l_p = \alpha(E)$.

Then we show

$$\frac{P_{\alpha(E)}(-z)}{\sin \pi \alpha(E)} = \sum \frac{P_l(\cos\theta)}{E - E_l}\, \beta(E)$$

Proof : $\doteq \sum_l P_l(z)\, u(E,l)$

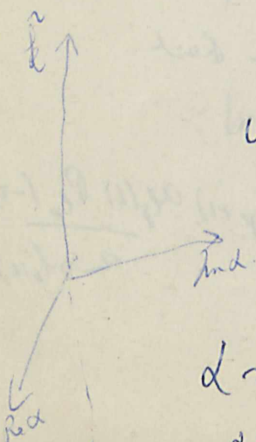
$$u(E,l) = \frac{1}{\pi} \frac{(2l+1)}{(\alpha-l)(\alpha+l+1)}$$

at $E = E_l$ $\alpha(E) - l$ is an integer.

Expand around this point

$$\alpha(E) = \alpha(E_l) + (E - E_l)\,\alpha'(E_l).$$

$$u(E,l) = \frac{1}{\pi} \frac{2(l+1)}{(\alpha-l)(\alpha+l+1)} = \frac{1}{(E-E_l)\,\alpha'(E_l)}.$$



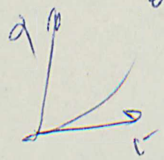
$\alpha' = \frac{d\alpha}{dE}$.

α real only if $k^2 < 0$ (bound state)

α is complex for $k^2 > 0$. (in this case we get resonances)

One can show

$$\frac{d\,\mathrm{Re}\,\alpha(E)}{dE} > 0$$ hence the $\alpha(E)$ curve is

rising 

$$\alpha(\alpha+1) = l(l+1) = \frac{-\int_0^\infty dr\,\chi\left(-\frac{\partial^2}{\partial r^2}+V-E\right)\chi}{\int_0^\infty \frac{\chi\chi}{r^2}\,dr}$$

$$\frac{\partial\,\alpha(\alpha+1)}{\partial E} = \frac{\int \chi\chi\,dr}{\int \frac{\chi\chi}{r^2}\,dr} \approx R^2 > 0.$$

Here a multitude of bound states is comprised in one term.

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limit $z \to \infty$

$$A(k,z) = \int + \frac{P_\alpha(-z)\,\beta(E)}{\sin \pi\alpha(E)}$$

$\downarrow$

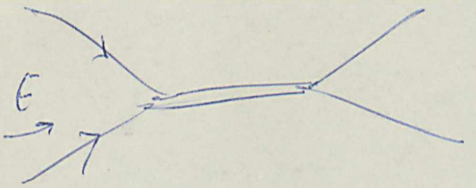
goes like $\frac{1}{\sqrt{z}}$

P_α goes like z^α

α : largest pole.

Mandelstam shows that $\frac{1}{z^l}$.

$z \to \infty$ $\qquad \sim \frac{z^\alpha\,\beta(E)}{\sin \pi\alpha(E)}.$

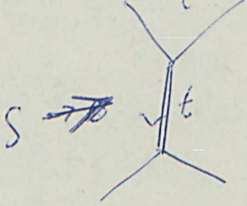


$s > 0$

$t < 0$: class of resonances.

In field theory take $t \to \infty$.

We have the inverse process. (no Regge to help)



peripheral model
particle exchange
is a crossed process

here s and t have been exchanged.

$z \to s$

we have $$\boxed{\frac{s^{\alpha(t)}\beta(t)}{\sin \pi\alpha(t)}}$$

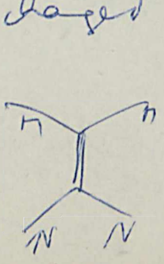
Regge picture
basic formula
for $s \to \infty$
high-energy limit
$= \sum \beta_l(\alpha) \dots$

bound states are exchanged between top and bottom: not a single bound state.

In the ordinary peripheral model, if a spin l particle is exchanged

$$\frac{P_l(\cos\theta)}{t - t_R} = \boxed{\frac{s^l \phi(t)}{t - t_R}}$$

has a fixed spin.



g, $g+2n$, $g+4n$ etc. are being exchanged.
so the whole class of bound states is being exchanged.

Consequences of new model. (instead of single particle exchange)

Gell-Mann conjecture

$\pi\pi$ $\quad \pi N$ $\quad NN$.

Differential cross section

$$\frac{d\sigma}{dt} = f(t) \quad s^{2\alpha(t)-2}$$

forward direction $t \to 0$ $\quad \frac{d\sigma}{dt} \approx f(0) \; s^{2[\alpha(0)-1]} \exp[t\alpha'(0)\log s]$

1) Diffraction ~~scattering~~ dominated by a peak that decreases logarithmically in width.

2) Tail of the peak fall exponentially.

3) Same constants $\alpha(0)$, $\alpha'(0)$ control all forward or backward scattering that relate to same quantum numbers.

So $\pi^+\pi^0$, K^+K^0 scattering are controlled by same $\alpha(0)$, $\alpha'(0)$ as for (np)

4) In contrast the peripheral picture gives α = const.

5) If Pomeranchuk is right $\sigma \sim$ const at $s \to \infty$.
then $\underline{\alpha(0) = 1}$. Hence there are α's with same quantum number as the vacuum except spin.
on $\alpha(0)$ is exchanged.

6) Still then

If $$\frac{d\sigma(s_1,t)/dt}{d\sigma(s_2,t)/dt} = \left(\frac{s_1}{s_2}\right)^{2(\alpha-1)}$$

varying energy

keeping t constant.

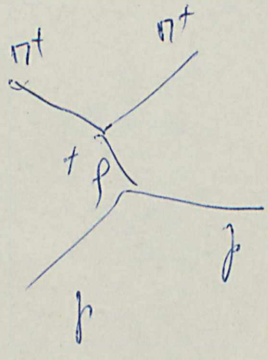
Experimental results.

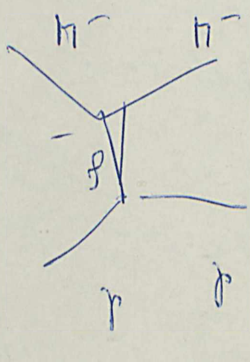
$\alpha(s)$ or $\alpha(m^2)$ experimentally.

Let $\alpha(0)=1$ (Pomeranchuk α).

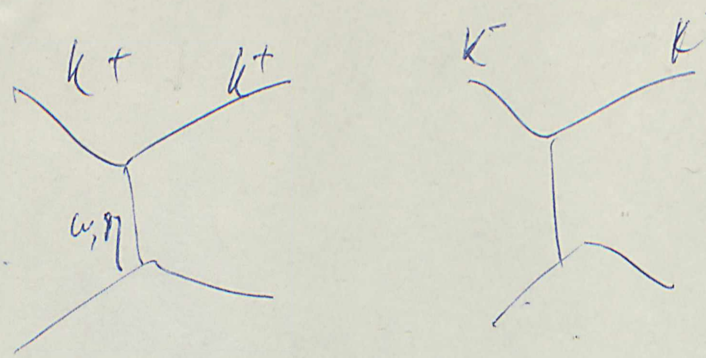
Mandelstam $\alpha_\rho(m^2)$ Re $\alpha_\rho(>50\ \mathrm{MeV}^2)=1$.

$\alpha_\omega(m^2)$ Re $\alpha(m_x^2=m_\omega^2)=1$.





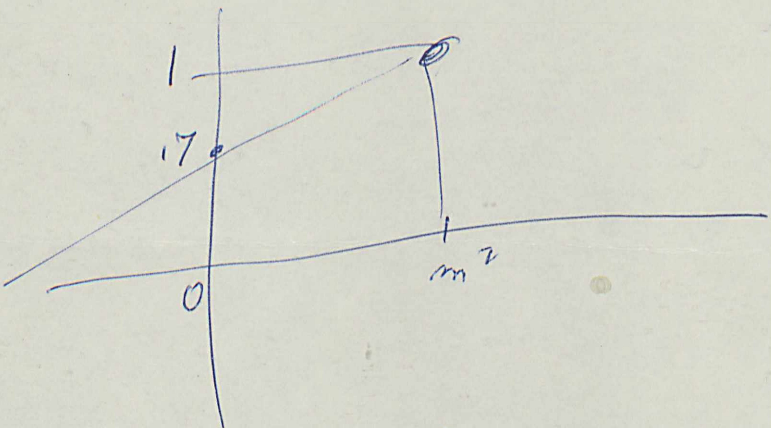
from Pomeranchuk $(\pi^+ p) = (\pi^- p)$ Only one α would fix apart. The difference comes from exchange of ρ. $s^{2[\alpha_\rho(t)-1]}$ would give the deviation from Pomeranchuk



P

One finds $\alpha_\rho(0) = .7$ $\alpha_\eta(0) = 2/3$.

Then draw curve (Chew).



$\alpha(8m_\pi^2) = 2$ Gell-Mann says $\alpha(1\,\text{BeV}) = 2$.

$\alpha_{\text{Pomeranchuk}}(t = -45m_\pi^2) = 1$.

This means the spin 2 object has mass $8m_\pi^2$.

Ford. Heavy Particles?

(μ-e/mass difference. (photon, W)

Propagators.

Bare particle $\varphi(x)|0\rangle = |1_x\rangle$ 1 bare particle created at x.

(undressed particle of Thing)

Z_3 (Renormalization constant of the field)

defined by $Z_3 = |\langle 1_{x=0} | p_n \rangle|^2$ for boson field

Renormalized φ.

$$\varphi_R = \frac{1}{\sqrt{Z_3}}\varphi .$$

Z_3 is the probability that the bare particle occurs in the physical particle state

Hence $Z_3 < 1$.

In Q.E. Renormalized charge

$$e^2 = Z_3 e_0^2$$

e_0 = bare charge

$e < e_0$.

Propagator:

Renormalized $\Delta'_F(x-y) \sim \langle 1_x | 1_y \rangle$ The prob. amplitude for 1 particle being created at x and annihilated at y.

$$\Delta'_F(k^2) = \frac{1}{k^2-\mu^2} + \int_{M_0^2}^{\infty} \frac{\sigma(m^2)\,dm^2}{k^2-m^2+i\varepsilon}$$

for a free field $\sigma = 0$.

μ^2: physical mass of the boson field

σ: probability that the physical particle propagates like a bare particle of mass m.

Sum rule for σ.

$$1 + \int \sigma(m^2)\, dm^2 = \frac{1}{Z_3}$$

No restriction placed on σ except that it is positive.

Now consider

$$\frac{1}{(k^2-\mu^2)\,\Delta'_F(k^2)} = 1 - \int_{M_0^2}^{\infty} \frac{G(m^2)\, dm^2}{k^2 - m^2 + i\epsilon}\,(k^2-\mu^2)$$

expected for.

In the high energy limit

$$\Delta'_F(k^2) \to \frac{1}{Z_3 k^2}$$

$$\frac{1}{(k^2-\mu^2)\,\Delta'_F(k^2)} \to Z_3 .$$

G must fall faster than $\frac{1}{m^2}$.

If Z_3 lies in the domain

Relation between G and σ

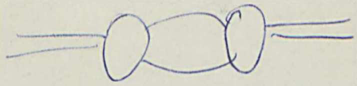
$$\sigma(m^2) = |\Delta'_F(m^2)|^2\, G(m^2)\,(m^2-\mu^2)^2$$

if $\sigma > 0$, $G > 0$.

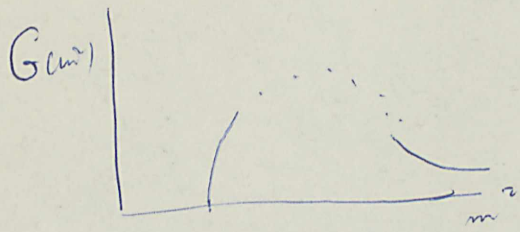
If substituted in Δ'_F one has an integral eq. for Δ' the solution of which is the expression for $\frac{1}{(k^2-\mu^2)\,\Delta'_F(k^2)}$

vertex intermediate state

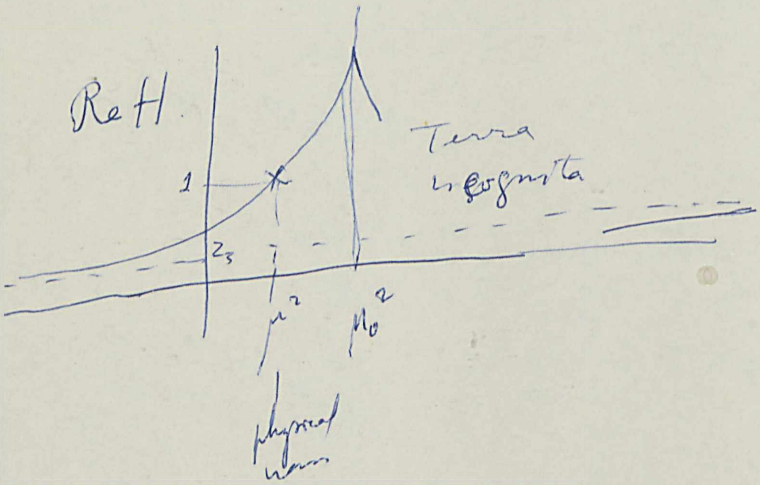
$$\Delta'_F = \Delta'_F \, \Gamma \, I \, \Gamma \, \Delta'_F$$



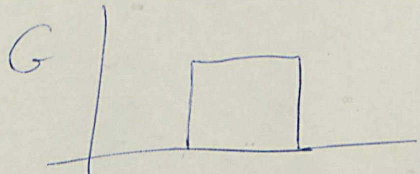
G has to fall off like $\frac{1}{m}$ and rises after being zero.



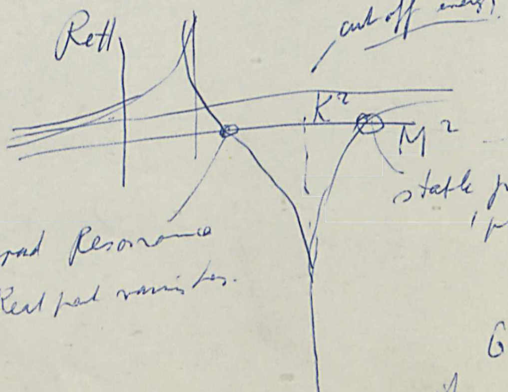
$$\text{Let } H = \frac{1}{(k^2-\mu^2)\,\Delta'_F(k^2)}$$



If one takes



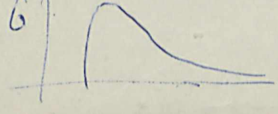
then



stable particle (pole). — has same quantum numbers as the original particle.

Broad Resonance
Real part vanishes.

If



Then instead of stable particle we have a sharp resonance.

approximate for small Z_3

$$\sigma(m^2) = C_M\,\delta(m^2 - M^2) + \sigma'(m^2)$$

$$C_M \sim \frac{1}{Z_3}$$

$$Z_3 + Z_3 C_M + Z_3 \int \sigma'(m^2)\, dm^2$$

$$M^2 = \frac{\langle m^2\rangle - \mu^2}{Z_3}\ ; \quad \langle m^2\rangle = \int m^2 \sigma(m^2)\, dm^2$$

$$\mu_0^2 \simeq M^2(1 + O(Z_3))$$

Ghost states.

If $0 < Z_3 < 1$ then no ghost states.

If Z_3 become < 0 (as in the Lee model for some value of the coupling constant) one gets ghost states. Z_3 is no longer a probability.

$Z_3 < 0$ in perturbation theory $\quad C_M \sim \infty$

Prob. for Higher state becoming large

Table II. Atomic and nuclear properties (dE/dx, collision mean free path, radiation length, etc.) of materials used as absorbers and detectors

Material	Z	A	Cross section σ [a] (barns)	$-\frac{dE}{dx}$ min [b] $\frac{\text{Mev}}{g/cm^2}$	Collision[a] length g/cm^2	L_{coll} cm	Radiation[c] length g/cm^2	L_{rad} cm	Density ρ (g/cm^3)
H_2	1	1.01	0.063	4.14	26.5	374	58	819.0	0.0708 {boiling at 1 atmos
Li	3	6.94	0.23	1.72	50.4	94.3	77.5	145	0.534 {boiling at 1 atmos
Be	4	9.01	0.28	1.71	55.0	30.5	62.2	34.6	1.8
C	6	12.00	0.33	1.86	60.4	39.0	42.5	27.4	1.55 (variable)
Al	13	26.97	0.57	1.66	79.2	29.3	23.9	8.86	2.70
Cu	29	63.57	1.00	1.45	105.4	11.8	12.8	1.44	8.9
Sn	50	118.70	1.55	1.27	129.7	17.8	8.54	1.17	7.30
Pb	82	207.21	2.20	1.12	156.2	13.8	5.8	0.51	11.34
U	92	238.07	2.42	1.095	163.6	8.75	5.5	0.29	18.7
Hydrogen (bubble chamber, -27.6^{o}K)				0.243 Mev/cm	26.5	452	58	990	0.0586
Propane (C_3H_8, bubble chamber)				0.935 Mev/cm	48.9	119.3	44.7	109.0	0.41
Polystyrene (CH scintillator)				2.14 Mev/cm	54.9	52.3	43.4	41.3	~ 1.05
Ilford emulsion				5.49 Mev/cm	103	27.0	11.2	2.91	3.815

Table III. Multiple Coulomb scattering and Lorentz transformation

The rms projected angle θ due to multiple Coulomb scattering (only) of a particle of charge z, momentum P, velocity V is

$$\theta_{proj} = z\,\frac{15(\text{Mev})}{PV(\text{Mev})}\sqrt{\frac{L}{L_{(rad)}}}\,(1+\epsilon)\ \text{radians};$$

L = Length in scatterer; L(radiation) from Table II. For $L \geqslant 1/10\ L(rad)$ ϵ is generally < 1/10. The distribution of θ is not truly Gaussian. The rms projected displacement is

$$y_{rms} = L\,\theta_{proj}/\sqrt{3}.$$

Lorentz transformations. Notation: Lower-case type for c.m. 4-momentum $(\vec{p}, w)$ and capitals for lab $(\vec{P}, W)$. (c≡1.) To transform from c.m. to lab write

$$\begin{pmatrix} \gamma & 0 & 0 & \eta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \eta & 0 & 0 & \gamma \end{pmatrix}\begin{pmatrix} p\cos\theta \\ p\sin\theta \\ 0 \\ w \end{pmatrix} = \begin{pmatrix} \gamma p\cos\theta + \eta w \\ p\sin\theta \\ 0 \\ \eta p\cos\theta + \gamma w \end{pmatrix} = \begin{pmatrix} P\cos\Theta \\ P\sin\Theta \\ 0 \\ W \end{pmatrix}$$

If two particles (1 and 2) collide, the invariant "mass" μ of the system is given by

$$\mu^2 = (W_1 + W_2)^2 - (\vec{P}_1 + \vec{P}_2)^2,$$

$$\gamma = \frac{W_1 + W_2}{\mu}; \quad \eta = \left|\frac{\vec{P}_1 + \vec{P}_2}{\mu}\right| = \gamma\beta.$$

Write T for lab kinetic energy, t for c.m.; thus $\mu = m_1 + m_2 + t_1 + t_2 = m_1 + m_2 + Q$. If the target is at rest $(0, m_2)$ μ simplifies:

$$\mu^2 = (m_1 + m_2)^2 + 2T_1 m_2.$$

To get a threshold T_1, set μ = sum of masses of reaction products, then

$$[\Sigma m(\text{products})]^2 = (m_1 + m_2)^2 + 2T_1 m_2.$$

Other invariants are: $w_1 w_2 - p_1 p_2 \cos\theta_{12}$ and

$$\frac{1}{p}\,\frac{d^2\sigma}{d\omega dw}.$$

The max. lab angle that a particle of c.m. momentum p_i can have is given by

$$\sin\Theta_i = \frac{\eta_i}{\eta}\ \left(\eta_i = \frac{p_i}{m_i}\ \text{must be} < \eta\right);$$

If $\eta_i > \eta$, then of course Θ_i can be π.
Crawford's mnemonic for extending nonrelativistic formulas to relativistic case: "To the rest energy of each moving particle add Q/2" where Q = the total kinetic energy (c.m.) = $\mu - \Sigma m_i$. Thus in the rest frame of a two-body decay the kinetic energy Q is shared between the two particles according to

$$t_1 = Q\,\frac{m_2 + Q/2}{\mu}, \quad t_2 = Q\,\frac{m_1 + Q/2}{\mu}.$$

The above of course applies in the c.m. for the production of a two-body final state. To express t in terms of p, apply the mnemonic to a single particle (then Q = t). The non-rel. relation $p^2 = 2tm$ becomes

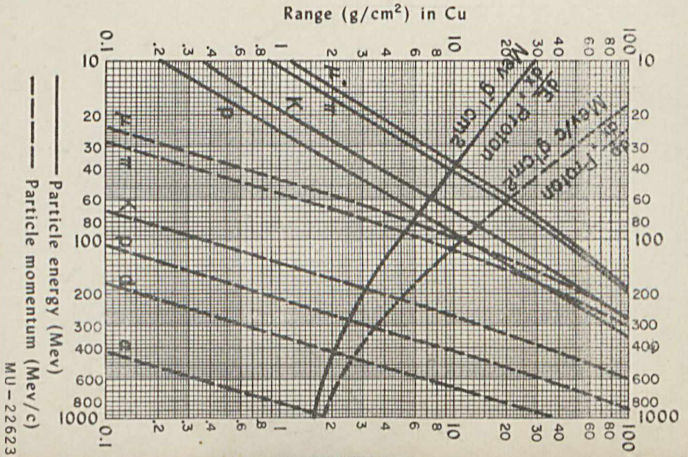
$$p^2 = 2t(m + t/2) = 2tm + t^2.$$

Energy Transfer in elastic collisions of beam (P_1, W_1) with resting target $(0, m_2)$, is

$$T_2 = 2m_1\,\frac{P_1^{\,2}}{\mu^2}\,\sin^2(\theta_{c.m.}/2).$$

Note that for max T_2, $\theta_{c.m.} = \pi$, so

$$T_{2max.} = 2m_1\,P_1^{\,2}/\mu^2$$



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TABLES FROM UCRL-8030(rev.). Table I. Masses and mean lives of elementary particles

(The antiparticles are assumed to have the same spins, masses, and mean lives as the particles listed)

	Particle	Spin	Mass (Errors represent standard deviation) (Mev)			Mass difference (Mev)			Mean life (sec)	
Photon	γ	1	0		γ	... ...		γ	Stable	
Leptons	ν	1/2	0		ν	... ...		ν	Stable	
	$e^{\mp}$	1/2	0.510976 ± 0.000007	(a)	$e^{\mp}$	... ...		$e^{\mp}$	Stable	
	$\mu^{\mp}$	1/2	105.655 ± 0.010	(b)	$\mu^{\mp}$, $\pi^{\pm}$	33.93 ± 0.05	(x)	$\mu^{\mp}$	$(2.212 \pm 0.001) \times 10^{-6}$	(r)
Mesons	π^{+}	0	139.59 ± 0.05	(*)	$\pi^{\pm}$, π^0	4.59 ± 0.01	(j)	$\pi^{\pm}$	$(2.55 \pm 0.03) \times 10^{-8}$	(w)
	π^0	0	135.00 ± 0.05	(*)				π^0	$(2.2 \pm 0.8) \times 10^{-16}$	(d)
	$K^{\pm}$	0	493.9 ± 0.2	(k)	K^+, K^0	3.9 ± 0.6	(i)	K^+	$(1.224 \pm 0.013) \times 10^{-8}$	(h)
	K^0, K_1, K_2	0	497.8 ± 0.6	(i)				K^0	50% K_1, 50% K_2	
					K_1, K_2	$(1.5 \pm 0.5)\hbar/\tau(K_1)$	(z)	K_1	$(1.00 \pm 0.038) \times 10^{-10}$	(e)
								K_2	$6.1(+1.6/-1.1) \times 10^{-8}$	(c)
Baryons	p	1/2	938.213 ± 0.01	(a)	p, n	1.2939 ± 0.0004	(t)	p	Stable	
	n	1/2	939.507 ± 0.01	(t)				n	$(1.013 \pm 0.029) \times 10^{3}$	(y)
	Λ	1/2	1115.36 ± 0.14	(v)	Λ	... ...		Λ	$(2.51 \pm 0.09) \times 10^{-10}$	(u)
	Σ^+	1/2	1189.40 ± 0.20	(l)	Σ^+, Σ^-	6.56 ± 0.22	(n)	Σ^+	$0.81(+0.06/-0.05) \times 10^{-10}$	(m)
	Σ^-	1/2	1195.96 ± 0.30	(n)	Σ^-, Σ^0	4.45 ± 0.4	(p)	Σ^-	$1.61(+0.1/-0.09) \times 10^{-10}$	(o)
	Σ^0	1/2	1191.5 ± 0.5	(*)				Σ^0	$< 0.1 \times 10^{-10}$	(s)
	Ξ^-	?	1318.4 ± 1.2	(f)	Ξ^-	... ...		Ξ^-	$1.28(+0.38/-0.30) \times 10^{-10}$	(f)
	Ξ^0	?	1311 ± 8	(q)	Ξ^0	... ...		Ξ^0	1.5×10^{-10} (1 event)	(q)

Walter H. Barkas, Arthur H. Rosenfeld, University of California, Berkeley, Sept. 1960.

Table IV. Atomic and nuclear constants in units of Mev, cm, and sec[a]

GENERAL ATOMIC CONSTANTS

$N = 6.0249 \times 10^{23}$ molecules/gram

$c = 2.99793 \times 10^{10}$ cm/sec

$e = 4.80286 \times 10^{-10}$ esu $= 1.6021 \times 10^{-19}$ coulomb.

1 Mev $= 1.6021 \times 10^{-6}$ erg [1 ev $= e(10^8/c)$]

$\hbar = 6.5817 \times 10^{-22}$ Mev sec $= 1.054 \times 10^{-27}$ erg sec.

$\hbar c = 1.9732 \times 10^{-11}$ Mev cm [$= \lambda$ for p = 1 Mev/c]

$k = 8.6167 \times 10^{-11}$ Mev/°C [Boltzmann constant]

$\alpha = \frac{e^2}{\hbar c} = 1/137.037$; $e^2 = 1.44 \times 10^{-13}$ Mev cm

Cross Section

$\sigma_{\text{Thompson}} = \frac{8}{3} \pi r_e^2 = 0.6652 \times 10^{-24} \text{cm}^2 = 0.6652$ barn

Magnetic Moment and Cyclotron Angular Frequency

$\mu_{\text{Bohr}} = \frac{e\hbar}{2mc} = 0.57883 \times 10^{-14}$ Mev/gauss

$\frac{1}{2}\omega_{\text{cyclotron}} = \frac{e}{2mc} = 8.7945 \times 10^6$ rad sec^{-1}/gauss

$g_{\text{electron}} = 2[1 + \frac{\alpha}{2\pi} - 0.328 (\frac{\alpha}{\pi})^2] = 2[1.0011596]$[b]

$g_{\text{muon}} = 2[1 + \frac{\alpha}{2\pi} + 0.75 (\frac{\alpha}{\pi})^2] = 2[1.001165]$[b]

QUANTITIES DERIVED FROM THE ELECTRON MASS, m

Mass and Energy

$m = 0.510976$ Mev $= 1/1836.12\ m_p = 1/273.26\ m_\pi$

Rydberg, $R_\infty = \frac{me^4}{2\hbar^2} = mc^2 \times \frac{\alpha^2}{2} = 13.605$ ev

Length (1 fermi $= 10^{-13}$ cm; 1 A $= 10^{-8}$ cm)

$r_e = e^2/mc^2 = 2.81785$ fermi

$\lambda_{\text{Compton}} = \frac{\hbar}{mc} = r_e \alpha^{-1} = 3.8612 \times 10^{-11}$ cm

$a_{\infty\,\text{Bohr}} = \frac{\hbar^2}{me^2} = r_e \alpha^{-2} = 0.52917$ A

Hydrogen-like atom (Non. Rel.; $\mu \equiv$ reduced mass).

$E_n = \frac{1}{2} \frac{\mu z e^4}{(n\hbar)^2}$; $a_{n=1} = \frac{\hbar^2}{\mu z e^2}$; $\left.\frac{v}{c}\right)_{\text{rms}} = \frac{ze^2}{n\hbar c}$

QUANTITIES DERIVED FROM THE PROTON MASS, m_p

Rest mass $= 938.211$ Mev/c^2 $= 1836.12\ m_e = 6.719\ m_\pi$

$= 1.007593\ m_1$

where $m_1 = 1$ amu $= \frac{1}{16}\ O^{16} = 931.141$ Mev.

Magnetic Moment and Cyclotron Angular Frequency

$\mu_p = \frac{e\hbar}{2m_p c} = 3.1524 \times 10^{-18}$ Mev/gauss

$\frac{1}{2}\omega_{\text{cyclotron}} = \frac{e}{2m_p c} = 4.7896 \times 10^3$ rad sec^{-1}/gauss

$\left.\frac{\mu}{\mu_p}\right)_{\text{proton}} = 2.79275$; $\left.\frac{\mu}{\mu_p}\right)_{\text{neutron}} = -1.9128$

Table IV (continued)

QUANTITIES DERIVED FROM THE MASS OF THE CHARGED PION, m_π

Rest mass $= 139.63$ Mev/c^2 $= 273.26\ m_e = 0.14882\ m_p$

Length

$\frac{\hbar}{m_\pi c} = 1.4132$ fermi ($\sim \sqrt{2}$ fermi)

Natural (= "geometrical") Nucleon Cross Section

$\pi \left(\frac{\hbar}{m_\pi c}\right)^2 = 62.7344$ mb (1 mb $= 10^{-27}$ cm^2)

(3/2, 3/2)πp Resonance of mass 1237 Mev (Q = 159 Mev).

Center-of-mass momentum: $p_\pi = 230$ Mev/c

Lab-system momentum: $P_\pi = 303$ Mev/c ($T_\pi = 195$ Mev)

RADIOACTIVITY

1 curie $= 3.7 \times 10^{10}$ disintegrations/sec

1 r $= 87.8$ ergs/g air $= 5.49 \times 10^7$ Mev/g air

Fluxes (per cm^2) to liberate 1 r in carbon:

3×10^7 minimum ionizing singly charged particles
0.9×10^9 photons of 1 Mev energy.

(These fluxes are actually correct to within a factor of two for all materials.)

Natural background: 100 mr/year

"Tolerance" 100 millirem/week [Note, 1 r may produce up to 10 "rem" (r equivalent for man), depending on type of radiation.]

MISCELLANEOUS

Physical Constants

1 year $= 3.1536 \times 10^7$ sec ($\approx \pi \times 10^7$ sec)

Density of air $= 1.205$ mg/cm^3 at 20°C

Acceleration by gravity $= 980.67$ cm/sec^2

1 calorie $= 4.184$ joules

1 atmosphere $= 1033.2$ g/cm^2

Numerical Constants

1 radian $= 57.29578$ deg; $e = 2.71828$

ln 2 $= 0.69315$; $\log_{10} e = 0.43429$;

ln 10 $= 2.30259$; $\log_{10} 2 = 0.30103$.

Stirling's approximation

$\sqrt{2\pi n} (\frac{n}{e})^n < n! < \sqrt{2\pi n} (\frac{n}{e})^n (1 + \frac{1}{12n-1})$

Gaussianlike Distributions

For $n > -1$ but not necessarily integral:

$$\int_0^\infty x^{2n+1} \exp\left[-\frac{x^2}{2\sigma^2}\right] dx = 2^n\, n!\, \sigma^{2n+2};\ (\tfrac{1}{2})! = \sqrt{\pi}/2$$

Relation between standard deviation σ and mean deviation α:

$2\sigma^2 = \pi\alpha^2$; $\sigma = 1.4826$ probable error.

Odds against exceeding one standard deviation = 2.15:1; two, 21:1; three, 370:1; four, 16,000:1; five, 1,700,000:1

[a] Based mainly on Cohen, Crowe, and Dumond, The Fundamental Constants of Physics (Interscience, New York, 1957), not on the later corrections of Cohen and Dumond, Phys. Rev. Lett. 1, 291 (1958).

[b] C. Sommerfield, Phys. Rev. 107, 328 (1957) and A. Petermans, Helv. Phys. Acta. 30, 407 (1957).

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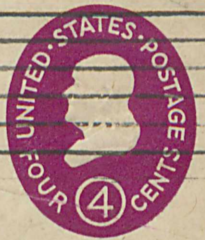

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