①

P. S. Physics

I thought you might be interested in what we are doing (Gell-Mann, Michel, Lévy, me)

We have been working on the axial vector current in $\Delta S=0$ decays, from the point of view of renormalizing $-\frac{G_A}{G}$ etc.

I would say that our principle results to the moment are

1.) $\partial_\mu J_{\mu 5}=0$ ← axial current does **not** generally imply $-G_A/G = 1$ unless you take the mass of the **nucleon** equal zero. There are two cases, assuming $\partial_\mu J_{\mu 5}=0$ {Bernstein, Gell-Mann, Michel}

1.) $M_N=0$ and $m_\pi \neq 0 \Rightarrow -\frac{G_A}{G}=1$

2.) $m_\pi=0$, $M_N \neq 0 \Rightarrow -\frac{G_A}{G} = \frac{S_{FC}^{-1}(i\gamma P = +M_N)}{2M_N}$

$$\cancel{= \int \frac{2m\,M_N}{i\gamma P + M_N} + \int \frac{F(M)\,dM}{im + M}}$$

So in this case (I keep writing m for M_N)

$$\frac{1}{-\frac{G_A}{G}} = 1 + \int \frac{F(M)\,dM}{m+M}\,2m + \int \frac{g(M)\,dM}{m-M}\,2m$$

↑ pair terms which can make $\frac{G_A}{G} > 1$

We have used the "Ward" identity

$$\left\{ S_{FC}^{-1}(P)\,\gamma_5\vec{\tau} + \gamma_5\vec{\tau}\,S_{FC}^{-1}(P+k) = i\,k_\mu\left(-\frac{G_A}{G}\right)\Gamma_{5\mu} \right\}$$

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(2)

This reduces to the identity on the prev. page when $a=0$.

2.) In any theory in which $\partial_\mu J_{\mu 5} = i a \pi$ where π is the π meson field we may derive the super renorm. ward identity (Bernstein, Gell-Mann, Michel)

nuc. propagator

$$S_{Fc}^{-1}(p)\vec{\tau}\,\gamma_5 + \vec{\tau}\,\gamma_5\, S_{Fc}^{-1}(p+k) = i k_\mu \left(-\frac{G_A}{G}\right) \Gamma_{\nu\mu 5}(p, p+k)$$

$$+\left(\sqrt{z_3}\, a\right) g_1\, D_{Fc}\, \Gamma_{5\nu}(p, p+k)$$

all quantities are renormalized

the axial vertex

the meson nucleon vertex

meson nucleon coupling const

renormalized meson propagator

and we may thus connect the $\pi \to \mu + \nu$ life time to the weak coupling constant by the equation (Feynman-Gell-mann)

$$\Gamma_\pi = \left(16\pi^2 \frac{g_1^2}{4\pi}\right)^{-2} \left(G_A m_N^2\right)^2 m_\pi \frac{m_\mu^2}{m_N^2}\left(1 - \frac{m_\mu^2}{m_\pi^2}\right)$$

$$\left[d_\pi(0)\, F_\pi(0)\right]^{-2}$$

these are form factors arising from the meson vertex and are of order $1 \pm 15\%$ from empirical the numbers for the quantities above.

and propagator; ie $D_{Fc} = \dfrac{1}{q^2 + m_\pi^2}\, d_\pi(q^2)$

$D_A(0) = \dfrac{d_\pi(0)}{\mu^2}$

There are two theories known to have currents of this form

1.) The Schwinger theory with the σ particle which is renormalizable and in which all corrections come out finite in all orders. We computed $-G_A/G$ in this theory and it is a series in g_1^2 the meson nucleon coupling constant, ~~[illegible]~~

2.) The ordinary pseudo-vector theory with the current

$$\vec{J}_{\mu 5} = \partial_\mu \pi \frac{1}{f_0} - \bar{\psi} \, i \vec{\tau} \gamma_\mu \gamma_5 \psi$$

this theory is not re-normalizable, but is apparently related to the Schwinger theory by a (non-unitary) tranformation. This is work of Gell-Mann and Lévy of which I have not seen the details. also Gell-Mann and Lévy are working on the problem of how to add on strange particles without destroying the beautiful properties of a current with $\partial_\mu J_{\mu 5} = i a \pi$, ie, that it relates the pion life time to G_A/G.

{ Pardon the horrible scrawl, but will keep you posted of how we are coming along }

Show that the weak interactions can be mediated by two vector fields $\vec{W}_\mu$ and $\vec{u}_\mu$.

$\frac{n^+}{\sqrt{2}} + \frac{\bar\Lambda}{2} p$

$-\frac{n^0}{2} + \frac{\bar\Lambda}{2}\frac{\Xi^0 - n}{2}$

$-\frac{n^0}{2} + \frac{\bar\Lambda}{2}\frac{\Xi^0 - n}{2}$

$-\frac{n^-}{\sqrt{2}} + \frac{\bar\Lambda}{2}\Xi^-$

Components of a vector

if $\binom{\psi_1}{\psi_2}$ is a spinor

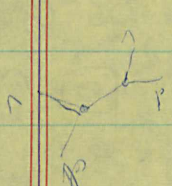
$\psi_1^2,\ \psi_2^2,\ \psi_1\psi_2$

$\binom{\psi_1}{\psi_2}\ \psi_1^*\ \psi_2^*$ $\left\{\begin{array}{l}\psi_1\psi_1^* - \psi_2\psi_2^* \\ \psi_1\psi_2^* \\ \psi_2\psi_1^*\end{array}\right.$

$$L_W = \overbrace{\left(\frac{n^+}{\sqrt{2}} + \frac{\bar\Lambda p}{2}\right)}^{W^+}\left(\frac{n^-}{\sqrt{2}} + \frac{\bar p \Lambda}{2}\right) + 2\overbrace{\left(-\frac{n^0}{2} + \frac{\bar\Lambda}{2}\frac{\Xi^0 - n}{2}\right)}^{(W^0 + iu^0)}\left(-\frac{n^0}{2} + \frac{\bar\Xi^0 - \bar n}{2}\Lambda\right)$$

$$- i\overbrace{\left(\frac{n^-}{\sqrt{2}} + \frac{\bar\Lambda}{2}\Xi^-\right)}^{u^-}\left(\frac{n^+}{\sqrt{2}} + \bar\Xi\frac{\Lambda}{2}\right)$$

$$= \frac{n^+ n^-}{2} + \frac{\bar\Lambda p}{2}\frac{\bar p \Lambda}{2} +$$



$$W^+W^- + W^0 +$$

$$L = \left(\frac{n^+}{\sqrt{2}} + \frac{\bar\Lambda p}{2}\right)W^- + 2\left(\frac{n^0}{\sqrt{2}} + \frac{\bar\Lambda}{2}\frac{\Xi^0 - n}{2}\right)(W^0 + iu^0)$$

$$+ \left(\frac{n^-}{\sqrt{2}} + \frac{\bar\Lambda}{2}\Xi^-\right)u^+ .$$

$J^0 J^{0\dagger}$

$= (J^0 + J^{0\dagger} + i(J^0 - J^{0\dagger}))$

$$J^+ J^{+\dagger} + J^0 J^{0\dagger} + J^- J^{-\dagger}$$ $\vec{J}$ is a complex vector.

$$J^+ J^{+\dagger} + (J^0_1 + iJ^0_2)(J^0_1 - iJ^0_2) + J^- J^{-\dagger}$$

$$= J^+ J^{+\dagger} + J_1^{0\,2} + J_2^{0\,2} + J^- J^{-\dagger}$$

$$(\bar\Lambda n)\left(W^0 + \frac{iu^0}{2}\right) + (\bar n \Lambda)\left(W^0 - \frac{iu^0}{2}\right).$$

$$= \frac{\bar\Lambda n + \bar n \Lambda}{2} W^0 + i\frac{\bar\Lambda n - \bar n\Lambda}{2} u^0$$

(use $(\bar\Lambda n + \bar n \Lambda)(\bar\Lambda n + \bar n \Lambda) + (\bar\Lambda n - \bar n \Lambda)(\bar\Lambda n - \bar n \Lambda) = \bar\Lambda n \bar\Lambda n + \bar n \Lambda \bar n \Lambda + 2\,\bar\Lambda n(\bar n \Lambda)$

$- \bar\Lambda n \bar\Lambda n \ldots$

16 matrices

$1, \ \vec{\rho}\cdot\vec{\sigma}, \ \vec{\rho}+\vec{\sigma}, \ (\vec{\rho}-\vec{\sigma}, \ \vec{\rho}\times\vec{\sigma}), \ \frac{\vec{\rho}\vec{\sigma}+\vec{\sigma}\vec{\rho}}{2} - \frac{1}{3}\vec{\rho}\cdot\vec{\sigma}$

$1 \quad 1 \quad 3 \quad 3 \quad 3 \quad 5$

anti-symmetric $\quad \vec{\rho}-\vec{\sigma}+i\vec{\rho}\times\vec{\sigma}$ $\quad (\vec{\rho}+i\vec{\sigma})\times(\vec{\rho}+i\vec{\sigma})$

also $\quad \vec{\rho}-\vec{\sigma}-i\vec{\rho}\times\vec{\sigma}$. $\quad = \vec{\rho}\times\vec{\rho} - \vec{\sigma}\times\vec{\sigma} + i(\vec{\rho}\times\vec{\sigma} + \vec{\sigma}\times\vec{\rho})$

orthogonal. $\quad (\vec{\rho}+i\vec{\sigma})\times(\vec{\rho}-i\vec{\sigma}) = \vec{\rho}\times\vec{\rho} + \vec{\sigma}\times\vec{\sigma} - 2i\vec{\rho}\times\vec{\sigma}$

$$\begin{pmatrix} 0 & 0 \\ ia & 0 \\ ia & 0 \\ b & -ia \end{pmatrix}$$

$\frac{1-\varepsilon}{2}\vec{\rho} - \frac{1-\varepsilon}{2}\vec{\sigma} \quad \frac{1-\varepsilon}{2}(\rho-\sigma)$

$\sigma_1 = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}$

$\sigma_1\sigma_2 = i\sigma_3$

$\begin{pmatrix} & 1 \\ 1 & \end{pmatrix}\begin{pmatrix} & -i \\ i & \end{pmatrix} = \begin{pmatrix} i & \\ & -i \end{pmatrix}$

$\vec{\rho}\times\vec{\sigma}\cdot\vec{\Pi} = (\rho_1\sigma_2 - \rho_2\sigma_1)\Pi_3 + (\rho_2\sigma_3 - \rho_3\sigma_2)\Pi_1 + (\rho_3\sigma_1 - \rho_1\sigma_3)\Pi_2 \quad -i\sigma_3(\Pi_1 - i\Pi_2)$

$$\begin{pmatrix} & \sigma_2+i\sigma_1 \\ \sigma_2-i\sigma_1 & \end{pmatrix}\Pi_3 + \begin{pmatrix} -\sigma_2 & -i\sigma_3 \\ i\sigma_3 & +\sigma_2 \end{pmatrix}\Pi_1 + \begin{pmatrix} \sigma_1 & -\sigma_3 \\ -\sigma_3 & -\sigma_1 \end{pmatrix}\Pi_2$$

$\sigma_1\Pi_2 - \sigma_2\Pi_1 = \frac{1}{2i}(\sigma_1 + i\sigma_2)(\Pi_1 - i\Pi_2) = \sigma_1\Pi_1 + \sigma_2\Pi_2 + i\sigma_2\Pi_1 + i\sigma_1\Pi_2 = i(\sigma^-\Pi^+ - \Pi^-\sigma^+)$

$i(\sigma_1 - i\sigma_2)$
$i\sigma^-$
$-i(\sigma_1+i\sigma_2)$

$$= \begin{pmatrix} & i\sigma^-\Pi_3 \\ -i\sigma^+\Pi_3 & \end{pmatrix} + \begin{pmatrix} i(\sigma^-\Pi^+ - \Pi^-\sigma^+) & -i\sigma_3\Pi^- \\ i\sigma_3\Pi^+ & +i(\sigma^-\Pi^+ - \Pi^-\sigma^+) \end{pmatrix}$$

$\rho_3 + \sigma_3 = \begin{pmatrix} 1 & & & \\ & 0 & & \\ & & 0 & \\ & & & -1 \end{pmatrix}$

$\rho^+ + \sigma^+ \quad \begin{pmatrix} \sigma^+ & \\ 1 & \sigma^+ \end{pmatrix}$

$\rho_+ + \sigma^-$

$$\vec{\rho}\times\vec{\sigma}\cdot\vec{\Pi} = \begin{pmatrix} -i(\sigma^-\Pi^+ - \Pi^-\sigma^+) & i(\sigma^-\Pi_3 - \sigma_3\Pi^-) \\ -i(\sigma_3\Pi^+ - \sigma^+\Pi_3) & i(\sigma^-\Pi^+ - \Pi^-\sigma^+) \end{pmatrix}$$

ρ real $+ 3 = 8$

$-\frac{1}{6}a^+b^- + \frac{1}{6}a^-b^+$
$-\frac{1}{6}a^+b^- + \frac{1}{6}a^-b^+$

$$= \begin{pmatrix} \Pi^- & \Pi^+ & \Pi^- \\ & \Pi_3 & \Pi^- \\ & 0 & \Pi^+ \\ & \Pi^- & 0 \end{pmatrix}$$

$\Pi_1 \quad \Pi_3 \quad \Pi_1$

$\varepsilon = \frac{1+\rho\cdot\sigma}{2} \quad 2\varepsilon - 1 = \rho\cdot\sigma$.

$$\varepsilon = \begin{pmatrix} 1 & & & \\ & & 1 & \\ & 1 & & \\ & & & 1 \end{pmatrix} \qquad 1 = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

$\sigma_2 = \begin{pmatrix} & i \\ -i & \end{pmatrix}$

$\sigma_1 - i\sigma_2$

$i\sigma_2 = \begin{pmatrix} & -1 \\ 1 & \end{pmatrix}$

$$\frac{1-\varepsilon}{2} = \frac{1}{2}\begin{pmatrix} 0 & & & \\ & \frac{1}{2} & -\frac{1}{2} & \\ & -\frac{1}{2} & \frac{1}{2} & \\ & & & 0 \end{pmatrix} \qquad \frac{1-\varepsilon}{2} =$$

$\frac{\sigma_1 + i\sigma_2}{2} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$

$\frac{\sigma_1 - i\sigma_2}{2} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$

$a^-b^+ \qquad \begin{pmatrix} & \\ & \end{pmatrix} \qquad \sigma$

$\rho_3(\sigma_1 + i\sigma_2)$

$$\begin{pmatrix} & & & \\ & -a & & \\ & -a & & \\ & & & +a \end{pmatrix}$$

$$\boxed{\frac{\rho^+\sigma^- + \rho^-\sigma^+}{2} - \rho_3\sigma_3}$$

$= \rho_1\sigma_1 + \rho_2\sigma_2 - \rho_3\sigma_3 = \vec{\rho}\cdot\vec{\sigma} -$

$\rho_1\sigma_+ a - \rho_3 i\sigma_3 a$

$(\rho_-\sigma_+ - \rho_3\sigma_3)a$

$-\rho_3\sigma_+ b$

$\frac{\rho_1 + i\rho_2}{\sqrt{2}} \cdot \frac{\sigma_1 - i\sigma_2}{\sqrt{2}} + \rho_1 - i\rho_2$

$\frac{\rho_1\sigma_1 + \rho_2\sigma_2}{2} + i -$

$(\frac{\rho_1 - i\rho_2}{2})(\frac{\sigma_1 + i\sigma_2}{2}) - \rho_3\sigma_3 = \frac{\rho_1\sigma_1 + \rho_2\sigma_2}{4} - \rho_3\sigma_3 + i(\rho_1\sigma_2 - \rho_2\sigma_1)a$

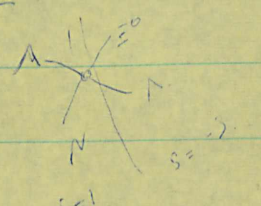
$\rho^+ \sigma^+$

ρ

$\left[\bar\Lambda(\Xi^0 - n)\right](\bar\Xi^0 - \bar n)\Lambda\}$

$(\bar\Lambda\Xi^0 - \bar\Lambda n)(\bar\Xi^0\Lambda - \bar n\Lambda) = \bar\Lambda\Xi^0\bar\Xi^0\Lambda + \bar\Lambda n\bar n\Lambda$

$\mp \bar\Lambda\Xi^0\bar n\Lambda - \bar\Lambda n\bar\Xi^0\Lambda$ ...



$-\bar\Lambda(\Xi^0\bar n + n\bar\Xi^0)\Lambda$

$\bar\Lambda(\Xi^0$

$(\bar\Lambda i\gamma_5\Xi^0 - \bar\Lambda n)(\bar\Xi^0\gamma_5\Lambda + \bar n\Lambda)$

$[(\bar\Lambda i\gamma_5\Xi^0)\bar n\Lambda] - (\bar\Lambda n)(\bar\Xi^0\gamma_5\Lambda)$

$= \Xi^{0\dagger} i\gamma_5\gamma_0\Lambda$

$= \bar\Xi^0 i\gamma_5\Lambda$

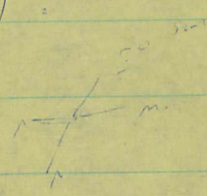
$(\bar\Lambda n)^\dagger = n$

$\Lambda^\dagger\gamma_0 n$.

$(\bar\Lambda n + \bar n\Lambda)w_0 + \frac{\bar\Lambda n - \bar n\Lambda}{i}u_0$

$(\bar\Lambda n + \bar n\Lambda)(\bar\Lambda n + \bar n\Lambda) \pm \left(\frac{\bar\Lambda n - \bar n\Lambda}{i}\right)\left(\frac{\bar\Lambda n - \bar n\Lambda}{i}\right) =$

$(\bar\Lambda\Xi^0 + \bar\Xi^0\Lambda)$



$(\bar\Lambda n \pm \bar\Xi^0\Lambda)(\bar n\Lambda - \bar\Lambda\Xi^0)$

$= \bar\Lambda n\bar n\Lambda - \bar\Xi^0\Lambda\bar\Lambda\Xi^0 \quad = \bar\Lambda n\bar\Lambda\Xi^0 - \bar\Xi^0\Lambda\bar n\Lambda$

$\rho_3\sigma \quad V_1 + V_2$

$\frac{1}{2}\rho_3 + \quad V_{3}$

Decomposition of the 4×4 matrix A

$$A = 1,\ \vec\rho\cdot\vec\sigma,\ \vec\rho,\ \vec\sigma,\ \rho_m\sigma_n + \rho_n\sigma_m - \frac{1}{3}\vec\rho\cdot\vec\sigma,\ \rho_m\sigma_n - \rho_n\sigma_m$$

Spin 0: Spin $0\ \frac{1}{2}\ 1\ \frac{3}{2}\ 2$

mult $2\ 0\ 3\ 0\ 1$ $\qquad 2+3\times3+5 = 2+9+5=16$

$(1+\vec\rho),\ \vec\sigma,\ \vec\rho\cdot\vec\sigma,\ \rho_m\sigma_n-\rho_n\sigma_m$

or

$$A = \begin{matrix}\rho^+ \\ \frac{1}{2}(1+\rho_3)\end{matrix},\ \begin{pmatrix}1+\rho_3 \\ \rho^-\end{pmatrix},\ \begin{pmatrix}\vec\rho\cdot\vec\sigma + \rho_1\sigma_2-\rho_2\sigma_1 \\ (\rho_2\sigma_3-\rho_3\sigma_2)+i(\rho_3\sigma_1-\rho_1\sigma_3)\end{pmatrix},\ \begin{pmatrix}(\rho_2\sigma_1-\rho_3\sigma_2)-i(\rho_3\sigma_1-\rho_1\sigma_3) \\ \vec\rho\cdot\vec\sigma-\rho_1\sigma_2+\rho_2\sigma_1\end{pmatrix},\ \vec\sigma,\ \rho_m\sigma_n+\rho_n\sigma_m-\frac{1}{3}\vec\rho\cdot\vec\sigma$$

$\frac{1}{2} \qquad \frac{1}{2} \qquad \frac{1}{2} \qquad \frac{1}{2}$

spin $0\ \frac{1}{2}\ 1\ \frac{3}{2}\ 2$

multiplicity $0\ 4\ 0\ 0\ 1$ $\qquad 1\times5+1\times3+4\times2$

$= 5+3+8 = 16.$

$\sigma\ \vec\rho\,\vec\sigma\ (1+\vec\rho) \quad \vec\sigma,\ \vec\rho\times\vec\sigma \quad \vec\rho\vec\sigma+\vec\sigma\vec\rho-\frac{1}{3}\vec\rho\cdot\vec\sigma$

$0 \quad \frac{1}{2} \quad \frac{1}{2} \quad 1 \quad 1 \quad 2$

$0\ \frac{1}{2}\ 1\ \frac{3}{2}\ 2$

$1\ 2\ 2\ 0\ 1$

$$\rho_2\sigma_1-\rho_3\sigma_2 = \begin{pmatrix}0 & i\sigma_1\\ -i\sigma_1 & 0\end{pmatrix} - \begin{pmatrix}\sigma_2 & \\ & -\sigma_2\end{pmatrix} = \begin{pmatrix}-\sigma_2 & i\sigma_1\\ -i\sigma_1 & +\sigma_2\end{pmatrix} \qquad \rho_3\sigma_1-\rho_1\sigma_3 = \begin{pmatrix}\sigma_1 & -\sigma_3\\ -\sigma_3 & -\sigma_1\end{pmatrix}$$

$$\rho_3\sigma_1-\rho_1\sigma_3+i(\rho_2\sigma_1-\rho_3\sigma_1) = \begin{pmatrix}\sigma_1-i\sigma_2 & -(\sigma_1+\sigma_3)\\ \sigma_1-\sigma_3 & i\sigma_2-\sigma_1\end{pmatrix} = \begin{pmatrix}\sigma^+ & -\sigma_1-\sigma_3\\ \sigma_1-\sigma_3 & -\sigma^+\end{pmatrix} \quad \vec\rho\cdot\vec\sigma = \begin{pmatrix}\sigma_3 & \sigma^-\\ \sigma^+ & -\sigma_3\end{pmatrix}$$

$$\vec\rho\cdot\vec\sigma - \rho_1\sigma_2+\rho_2\sigma_1 = \begin{pmatrix}\sigma_3 & \sigma^--i\sigma^+\\ \sigma^++i\sigma^- & -\sigma_3\end{pmatrix} \qquad \rho_1\sigma_2-\rho_2\sigma_1 = \begin{pmatrix} & \sigma_2+i\sigma_1\\ \sigma_2+i\sigma_1 & \end{pmatrix}$$

$-i(\sigma_1+i\sigma_2)$

$i(\sigma_1+i\sigma_2)$

$-\sigma_1+i\sigma_2$

$\frac{3}{2}$:

$\rho^+\sigma^+ = (\rho_1+i\rho_2)(\sigma_1+i\sigma_2) = \rho_1\sigma_1-\rho_2\sigma_2+i(\rho_1\sigma_2+\rho_2\sigma_1) = T_{11}-T_{22}+2iT_{12}$

$\rho^-\sigma^- = (\rho_1-i\rho_2)(\sigma_1-i\sigma_2) = \rho_1\sigma_1-\rho_2\sigma_2-i(\rho_1\sigma_2+\rho_2\sigma_1)$

$(1-\rho_1\sigma_1-\rho_2\sigma_2-\rho_3\sigma_3)\,\sigma_1$

$= \rho_1-\sigma_1+i(\sigma_2\rho_3-\rho_2\sigma_3)$

$$\vec T:\ \Big(\vec\sigma\ \overbrace{\frac{1-\vec\rho\cdot\vec\sigma}{2}}^{(1-\epsilon)}\Big) \qquad T_{\frac{1}{2}} = \frac{1-\vec\rho\cdot\vec\sigma}{2}\begin{pmatrix}\frac{\rho^+}{\sqrt2} = \frac{\rho_1+i\rho_2}{2}\\ \frac{1-\rho_3}{2}\end{pmatrix}$$

$\epsilon = \frac{1+\vec\rho\cdot\vec\sigma}{2}$

$1-\epsilon = 1-\frac{1+\vec\rho\cdot\vec\sigma}{2}$

$-i\rho_2\sigma_3+i\rho_3\sigma_2$

$$\sigma_1\left(\frac{1-\rho_1\sigma_1-\rho_2\sigma_2-\rho_3\sigma_3}{2}\right) = \frac{\sigma_1-\rho_1-\rho_2\sigma_1\sigma_2-\rho_3\sigma_1\sigma_3}{2} = \frac{\sigma_1-\rho_1}{2}+i\left(\frac{\rho_3\sigma_2-\rho_2\sigma_3}{2}\right)$$

$(\vec\sigma-\vec\rho+i\vec\rho\times\vec\sigma) \qquad (\vec\rho+\vec\sigma)_1+i(\vec\rho\times\vec\sigma)_2.$

unit

$1,\ \vec\rho\cdot\vec\sigma,\ \vec\rho+\vec\sigma,\ \vec\rho-\vec\sigma,\ \vec\rho\times\vec\sigma,\ \frac{1}{2}(\vec\rho\vec\sigma+\vec\sigma\vec\rho)-\frac{1}{3}\vec\rho\cdot\vec\sigma$

sym, sym, antisym, antisym, sym.

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σ^+ $\quad \rho^+$ $\quad (1+\rho)$

σ_3 $\quad (1+\rho_3)$ $\quad \rho^-$

σ^-

$\vec{\sigma}\cdot\vec{a} + i\rho_3(p_0 + i\vec{\sigma}\cdot\vec{b})$

$\vec{\rho}\cdot\vec{A} + \sigma_3(B_0 + i\vec{\rho}\cdot\vec{B})$

$\Lambda =$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{3}{2}$
	$(1+\rho_3)$	$(1+\sigma_3)$	$\rho_1 + i\rho_2$	$(\sigma_1+i\sigma_2)$	$\sigma^+\rho^+$	$\sigma^+(1+\rho_3)$
	$(\rho_1 - i\rho_2)$	$(\sigma_1 - i\sigma_2)$	$1-\rho_3$	$(1-\sigma_3)$	$\sigma^+(1+\rho_3)+\rho^+(1+\sigma_3)$	$\rho_3(1+\rho_3)+\sigma^+\rho^-$
					$\sigma^-(1+\rho_3)+\rho^-(1+\sigma_3)$	$\sigma^-(1+\rho_3)+\sigma_3\rho^-$
					$\sigma^+(1-\rho_3)+\sigma_3\rho^+$	$\sigma^-\rho^-$
					$\sigma^-\rho^+ + \sigma_3(1-\rho_3)$	
					$\sigma^-(1-\rho_3)$	

$1+\rho_3 \quad \rho^+ \qquad 1,\ i \qquad (\sigma^+, \sigma^0, \sigma^-) \times \frac{1}{i}$

$\rho^- \quad 1-\rho_3$

$\frac{1}{2} \qquad \frac{1}{2} \qquad 0,0 \qquad 1,1. \qquad \frac{3}{2} \qquad \frac{3}{2}$

complex number $\qquad$ complex vector

2 $\qquad$ 2 $\qquad$ 1 $\qquad$ 3 $\qquad$ 4 $\qquad$ 4

They are not all orthogonal.

$\rho^+ \quad \sigma^+$

$\rho_3 \quad \sigma_3$

$\rho^- \quad \sigma^-$

$$\frac{1+e}{2}\vec{\sigma}\frac{1-e}{2} = \frac{1+e}{2}\vec{\rho}\frac{1-e}{2} = \frac{1+e}{2}\frac{(\vec{\sigma}-\vec{\rho})}{2}\frac{1-e}{2}.$$

spin 2. $\qquad$ spin ?

out of 2 spin 1. $\qquad$ out of

$\rho^+\sigma^+$ $\qquad$ $\sigma^+(1+\rho_3)$.

$\rho_3\sigma^+ + \rho^+\sigma_3$

spin ? $\quad \rho^+\sigma^- + \rho^-\sigma^+ + \rho_3\sigma_3$ $\qquad 1,\ \rho,\ \sigma,\ (\rho_m\sigma_n + \rho_n\sigma_m) - (1+\vec{\rho}\cdot\vec{\sigma})$

$\rho_3\sigma^- + \rho^-\sigma_3$

$\rho^-\sigma^-$

$$\rho_m = \begin{pmatrix}\rho_1\\ \rho_2\\ \rho_3\end{pmatrix}(\sigma_1\ \sigma_2\ \sigma_3) = \begin{pmatrix}\rho_1\sigma_1 & (\rho_1\sigma_2+\rho_2\sigma_1) & \rho_1\sigma_3+\rho_3\sigma_1\\ & \rho_2\sigma_2 & \\ & & \rho_3\sigma_3\end{pmatrix}$$

$\rho^+\sigma^+(\rho_3\sigma^+ + \rho^+\sigma_3)\sigma_3$

Anything I must form a closed algebra $\qquad$ $T_r = \vec{\rho}\cdot\vec{\sigma}$

$a_0 + \bar{a}$ $\qquad 5+1 = 6$ units $\qquad (\rho^+)^2 = 0 \quad (\rho^+\sigma^+)^2 = 0$

spin 0. spin ? $\qquad (\rho_3\sigma^+)^2 = 0$

$\rho_3\sigma_3\sigma^+\rho^+$

1

$\rho_1 \quad \sigma_1 \qquad \rho_m\sigma_m$

$\rho_2 \quad \sigma_2 \qquad \rho$

16 $\quad \rho_3 \quad \sigma_3$

$\rho + 6 + 1$

$$\begin{pmatrix}\rho_1\sigma_1 - \frac{1}{3}\vec{\rho}\cdot\vec{\sigma} & \rho_1\sigma_2+\rho_2\sigma_1 & \rho_1\sigma_3+\rho_3\sigma_1\\ & \rho_2\sigma_2 - \frac{1}{3}\vec{\rho}\cdot\vec{\sigma} & \rho_2\sigma_3+\rho_3\sigma_2\\ & & \rho_3\sigma_3 - \frac{1}{3}\vec{\rho}\cdot\vec{\sigma}\end{pmatrix}$$

The trace is zero.

$1,\ \vec{\rho}\cdot\vec{\sigma},\ \vec{\rho},\ \vec{\sigma},\ (\rho_m\sigma_n + \rho_n\sigma_m - \frac{2}{3}\delta_{mn}\vec{\rho}\cdot\vec{\sigma}),\ (\rho_m\sigma_n - \rho_n\sigma_m)$

$0 \qquad 0 \qquad 1 \qquad 1 \qquad 2 \qquad 1$

$$f_p = \frac{1}{\sqrt{2p_0}} e^{ip\cdot x}$$

$$I = \sqrt{2q_0 2q_0'} \langle q' | j(0) | q \rangle = \sqrt{2q_0 2q_0'} \int d^4x \int d^4y\, f_{q'}^*(x) \{ $$

$$= \sqrt{2q_0 2q_0'} \int d^4x \int d^4y\, f_{q'}^*(x) \{ (\mu^2 - \Box_x)(\mu^2 - \Box_y) \langle 0 | (\phi(x)\, j(0)\, \phi(y))_+ | 0 \rangle f_q(y) \}$$

where

$$(\Box + m^2) W = j(x).$$

$$\langle 0 | T[W_\alpha(x) \phi(y) \phi(z')] | 0 \rangle.$$

$$\langle \cancel{pp'} | H_{int} | p p' \rangle = \langle$$

$$(\gamma_\mu \partial_\mu - m)\Psi = (\gamma_\mu \partial_\mu + m) e^{i\varphi} \psi.$$

$$\gamma_\mu \partial_\mu \psi = m e^{2i\gamma_5 \varphi} \psi + W_\mu \cancel{\ } \frac{1+\gamma_5}{2} \gamma_\mu \psi.$$

$$\gamma_\mu \partial_\mu (e^{i\gamma_5 \varphi} \psi) = m (e^{i\gamma_5\varphi} \psi) +$$

$$\gamma_\mu \partial_\mu (e^{-i\gamma_5 \varphi} \tilde\psi) = m e^{2i\gamma_5\varphi} e^{-i\gamma_5 \varphi} \tilde\psi + W_\mu \cancel{\ } \frac{1+\gamma_5}{2} \gamma_\mu e^{-i\gamma_5\varphi} \tilde\psi$$

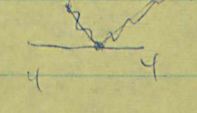
$$e^{-i\gamma_5 \varphi} \gamma_\mu \partial_\mu \tilde\psi + e^{i\gamma_5\varphi} i\gamma_5 (\gamma_\mu (\partial_\mu \varphi)) \tilde\psi = m e^{i\gamma_5\varphi} \tilde\psi + W_\mu \cancel{\ } \frac{1+\gamma_5}{2} \gamma_\mu e^{-i\gamma_5\varphi} \tilde\psi$$

$$W_\mu \to e^{2i\alpha} W_\mu$$

$$(\gamma_\mu \partial_\mu + m) \tilde\psi = i\gamma_5 \gamma_\mu (\partial_\mu \varphi) \tilde\psi + W_\mu e^{-i\gamma_5 \varphi} \cancel{\ } \frac{1+\gamma_5}{2} \gamma_\mu e^{-i\gamma_5\varphi} \tilde\psi$$

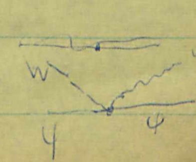
$$= i\gamma_5 \gamma_\mu (\partial_\mu \varphi) \tilde\psi + W_\mu e^{-i(1\pm\gamma_5)\varphi} \frac{1+\gamma_5}{2} \gamma_\mu \tilde\psi.$$

$$= e^{-2i\varphi} \frac{1+\gamma_5}{2} \gamma_\mu \tilde\psi.$$



$$\bar u^* \gamma_\mu u\, \cdots$$

$$(\gamma_\mu \partial_\mu + m)\psi = i\gamma_5 \gamma_\mu (\partial_\mu \varphi) \psi + \sqrt{G}\, W_\mu e^{-2i\gamma_5 \varphi} \frac{1+\gamma_5}{2} \gamma_\mu \psi.$$



inv. under $\psi \to \psi$ $\varphi \to \varphi + \alpha$ $W_\mu \to e^{2i\alpha} W_\mu$

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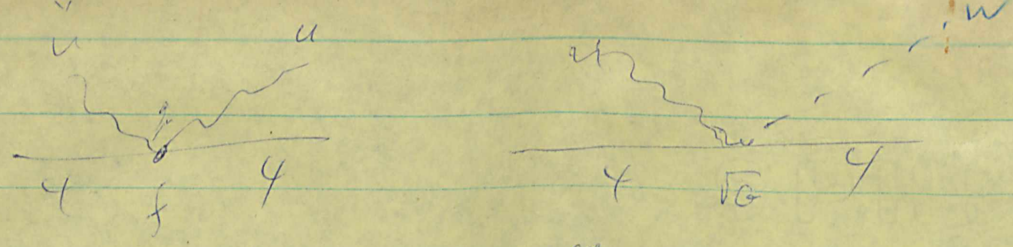
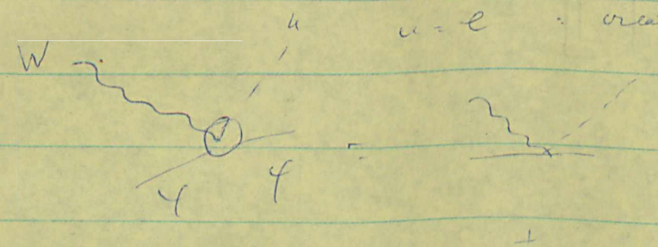
$W_\mu(\bar\psi\gamma_\mu\psi + \phi\partial_\mu\phi)$

$\Box\varphi = \bar\psi\gamma\psi$



$u = e^{2i\gamma\varphi}$ creates a pion or nothing



$n \to p + \pi + e + \bar\nu$

$W \to \psi\ \varphi.$

$\gamma_5(1 + \gamma_5\varphi + \varphi^2 + \dots)$

$(\partial\varphi)^2 \qquad L = W_\mu(\partial_\mu\varphi + \bar\psi\gamma_\mu\psi)$

$\gamma_5 + \frac{2\varphi}{2!} + \frac{3\gamma_5^2}{3!} + \dots$

$\Box\varphi =$

$= \bar\psi\psi + c\,\gamma_5\bar\psi\psi + \dots$

$(\partial\varphi)^2 \cdot \bar\psi\gamma\partial\psi + \bar\psi e^{2i\gamma\varphi}\psi + W_\mu(\bar\psi\gamma_\mu\frac{1+\gamma_5}{2}\psi + \partial_\mu\phi)$

$(M^2 + \Box)W_\mu = \bar\psi\gamma_\mu\frac{1+\gamma_5}{2}\psi + \partial_\mu\varphi.$

$\Box\varphi =$

16 - spin 2 : 5 components

spin $\frac{3}{2}$: 3 $\quad\psi_\mu$

1 $\quad\psi$

1×2 matrix or orthogonal V2 spinors = 1 spin $\frac{1}{2}$ or 1 spin 0 + 1 spin 1.

2×2 matrix : 2 spin $\frac{1}{2}$ or 1 spin 0 + 1 spin 1. vector
+1 spin 0 + 1 spin 1. axial vector

8 real or 4 complex parts.

4×4 matrix : 2 spin $\frac{3}{2}$ + 4 spin $\frac{1}{2}$. (Rarita-Schwinger)

16 complex. 8 complex 8 complex.

16 complex or 32 real

8 complex 2×4 matrix : 1 spin $\frac{3}{2}$ + 1 in 3 + 1 in 5. = 1 spin $\frac{3}{2}$ + 2 spin 0 + 2 spin 1.

A : 2 spin $\frac{3}{2}$ + 4 spin $\frac{1}{2}$.

or A : 2 spin $\frac{3}{2}$ + 2 spin $\frac{1}{2}$ + 2 spin 0 + 2 spin 1.

---

if $\mathrm{Tr} A = 0$, then A : 2 spin $\frac{3}{2}$ + 2 spin $\frac{1}{2}$ + 2 spin 1. The doubling is due to opposite parities.

$$A = \begin{pmatrix} a_0 + \vec{a} & c_0 + \vec{d} \\ b_0 + \vec{b} & -a_0 + \vec{b} \end{pmatrix} \qquad \begin{matrix} \Lambda N_1 \\ \Lambda N_2 \end{matrix} \qquad \begin{matrix} \vec{n}_1 \\ \vec{n}' \end{matrix} \qquad \begin{matrix} a_0 + \vec{a} + i\vec{a}' \\ -a_0 + \vec{a} - i\vec{a}' \end{matrix}$$